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International Council for the Exploration of the Sea

C. M. 1980/L: 18
Biological Oceanography Committee

Critical Scales for Phytoplankton Patchiness and Advection Diffusion Models

by

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Abstract

The problem of critical scales for phytoplankton patches is discussed taking into account the combined advection diffusion effect on the patch. Dependent on the spatial structur of the advection the critical length can vary rapidly with time.

Introduction

According to KIERSTAED and SLOBODKIN (KS-model, 1953) a phytoplankton patch disperses, if its spatial scale L is smaller than a critical scale L_c, otherwise, $L > L_c$, the patch increases in size with time. The critical length L_c is defined by

$$L_c = \pi \left(\frac{A}{\nu}\right)^{\frac{1}{2}} \quad (1)$$

Here A is the constant horizontal diffusivity, and ν is the rate of growth of plankton, respectively. It was mentioned by several authors (see e.g. OKUBO, 1978; here the reader may find an extensive list of references) that the KS-model is extremly simplified and some refining of both physical and biological aspects are necessary. From the physical point of view there are the following critical points:

1. The constant diffusivity is a very rough approximation
2. The boundary condition that the patch is surrounded by unsuitable water is rather arbitrary
3. The interpretation of L_c involves a problem. Since any patch of size $L < L_c$ will be destroyed, the question arises how a patch ever can reach a scale $L > L_c$

The present paper deals with the critical length problem on the base of an advection-diffusion model. The biological aspects are the same as in the KS-model.

The Model

We study the evolution of a plankton distribution $P(\vec{r},t)$ from a certain initial patch $P(\vec{r},0) = P_o(\vec{r})$. The depth H of the upper mixed layer may coincides with the depth of the enphotic zone. In the frame of the model $P(\vec{r},t)$ is a solution of the diffusion equation

$$(2) \quad \left[\frac{\partial}{\partial t} + \vec{v}\cdot\nabla - A\Delta_h - A_z\Delta_z - \nu\right] P(\vec{r},t) = 0$$

with an initial condition

$$P(\vec{r},0) = P_o(\vec{r})$$

and boundary conditions

$$\frac{\partial P}{\partial z} = 0 \quad \text{at } z = 0 \text{ (sea surface) and } \quad z = H$$

A and A_z are local horizontal and vertical diffusivities. $\vec{v}$ is the advection field

$$(3) \quad \vec{v} = \vec{v}_h(x,y) + \hat{\vec{v}}(z) = \begin{pmatrix} u_o + u_x x + u_y y \\ v_o + v_x x + v_y y \end{pmatrix} + \begin{pmatrix} \hat{u}(z) \\ \hat{v}(z) \end{pmatrix}$$

with

$$\operatorname{div} \vec{v} = 0 \; .$$

The linear horizontal advection $\vec{v}_h(x,y)$ is considered to be a mesoscale feature of a large scale circulation pattern. Introducing the streamfunction $\psi = u_y$, $\psi = -v_x$ we have with $u_y = g$, $v_x = -f$, $u_x = -v_y = h$

$$(4) \quad \psi = f\frac{x^2}{2} + g\frac{y^2}{2} - hxy + (u_o + \hat{u})y - (v_o + \hat{v})x$$

By means of $\delta = h^2 - fg$ the streamline pattern may be classified. For $\delta < 0$ it follows an eddy structure and for $\delta > 0$ we find a deformation field. Both examples are mesoscale features of long wave phenomenons. For large times $\beta_1 t \gg 1$ ($\beta_1 = \pi^2 A_z/H^2$ inverse vertical mixing time) the patch is vertically mixed and the solution can be found by means of a perturbation theory (FENNEL 1980a). Performing a Fouriertransformation

$$P(k,\varkappa,t) = \int dx\,dy\, e^{-ikx - i\varkappa y} P(x,y,t)$$

we have

$$(5) \quad P(k,\varkappa,t) \approx P_o(k,\varkappa)\, e^{-a\frac{k^2}{2} - b\frac{\varkappa^2}{2} - ck\varkappa - idk - ie\varkappa + \nu t}$$

with

$$(6) \quad a(t) = 2At + \frac{Ah}{\delta}\left[1 - ch(2\sqrt{\delta}t)\right] + \frac{At}{\delta}\left[2h^2 + g(g-f)\right]\left(\frac{sh\,2\sqrt{\delta}t}{2\sqrt{\delta}t} - 1\right) + F_x\left[\hat{u}(z)\right]t$$

$$(7) \quad b(t) = 2At + \frac{Ah}{\delta}\left[ch(2\sqrt{\delta}t) - 1\right] + \frac{At}{\delta}\left[2h^2 - f(g-f)\right]\left(\frac{sh\,2\sqrt{\delta}t}{2\sqrt{\delta}t} - 1\right) + F_y\left[\hat{v}(z)\right]t$$

$$(8) \quad c(t) = \frac{A}{2\delta}(g-f)\left(ch(2\sqrt{\delta}t) - 1\right) + \frac{Ah}{\delta}(f+g)\left(\frac{sh\,2\sqrt{\delta}t}{2\sqrt{\delta}t} - 1\right) + F_{xy}\left[\hat{u}(z), \hat{v}(z)\right]$$

a, b and c correspond to the dispersion of patches initially concentrated on a point ($P_o(\vec{r}) = \delta(x)\,\delta(y)$, $P_o(k,\varkappa) = 1$). d and e describe the movement of the patch along the streamlines.

$$(9) \quad d(t) = \frac{u_o}{\sqrt{\delta}}\, sh\sqrt{\delta}t + \frac{v_o g - u_o h}{\delta}\left(ch\sqrt{\delta}t - 1\right) + \frac{2}{H}\int_0^H dz\, \hat{u}(z)$$

$$(10)\quad e(t) = \frac{v_z}{\sqrt{\delta}} \, \mathrm{sh}\sqrt{\delta} t + \frac{v_0 h - u_0 f}{\delta} (\mathrm{ch}\sqrt{\delta} t - 1) + \frac{2}{H}\int_0^H dz \, \hat{v}(z)$$

The functionals F_x, F_y and F_{xy} are given by (see FENNEL 1980b)

$$(11)\quad F_x[\hat{u}(z)] = \frac{8\pi^2}{\beta_1 H^3}\int_0^H \left[\int_0^z \hat{u}(z')dz' - \frac{z}{H}\int_0^H \hat{u}(z')dz'\right]^2 dz$$

$$(12)\quad F_y[\hat{v}(z)] = \frac{8\pi^2}{\beta_1 H^3}\int_0^H \left[\int_0^z \hat{v}(z')dz' - \frac{z}{H}\int_0^H dz' \, \hat{v}(z')dz'\right]^2 dz$$

$$(13)\quad F_{xy}[\hat{u},\hat{v}] = \frac{8\pi^2}{\beta_1 H^3}\int_0^H dz \left(\int_0^z \hat{u}(z')dz' - \frac{z}{H}\int_0^H \hat{u}(z')dz'\right)\left(\int_0^z dz' \hat{v}(z') - \frac{z}{H}\int_0^H \hat{v}(z')dz'\right)$$

The distribution $P(k,\varkappa,t)$ given by (5) in connection with (6) to (13) looks rather complicated. But the essential point is the following: The dispersion a, b and c can vary more or less rapidly with time. That time dependence is controled by the spatial structur of the advection ($\delta = h^2 - gf$). In the case of an eddy structure ($\delta < 0$) a, b, c are proportional to t ($t \gg |\sqrt{\delta}|^{-1}$) but in the case of a deformation field ($\delta > 0$) these quantities vary with $e^{2\sqrt{\delta}t}$ for large times ($t \gg \delta^{-\frac{1}{2}}$).

The critical length

Based on (5) we discuss the time dependent wave number spectrum $P(k,\varkappa,t)$. For

$$ak^2 + b\varkappa^2 + ck\varkappa > \nu t$$

$P(k,\varkappa,t)$ decreases with time, while for

$$ak^2 + b\varkappa^2 + ck\varkappa < \nu t$$

the patch increases. Critical scales are determined by

$$(14)\quad ak^2 + b\varkappa^2 + ck\varkappa = \nu t$$

By rotation of the coordinate system $(k,\varkappa)$ into the principle axes system $(k^*, \varkappa^*)$ it follows

$$(15)\quad \frac{k^{*2}}{\frac{\nu t}{a^*}} + \frac{\varkappa^{*2}}{\frac{\nu t}{b^*}} = 1$$

where the principle axes are

$$a^* = \frac{1}{2}\left[a + b + \sqrt{(a-b)^2 + c^2}\right]$$

$$b^* = \frac{1}{2}\left[a + b - \sqrt{(a-b)^2 + c^2}\right]$$

Due to the anisotropic nature of the advection there a two critical wavenumbers, given by the minor and the major principle axis.

$$(16)\quad k_1 = \frac{2\pi}{L_1} = \sqrt{\frac{\nu t}{a^*(t)}} \quad , \quad k_2 = \frac{2\pi}{L_2} = \sqrt{\frac{\nu t}{b^*(t)}}$$

The interpretation of k_1 and k_2 is the following:
The small wavenumber part of the spectrum $P(k, \varkappa, t)$ that means $k \leq k_1, \varkappa \leq k_1$ will increase with time. The large wavenumber part, that means $k > k_2, \varkappa > k_2$ will decrease with time.

For wavenumers $k_1 < k < k_2$ and $k_1 < \varkappa < k_2$ it must be proved by mechs of (14) whether the corresponding spectral part increases or decreases.

The combined effect of advection and diffusion on the patch smoothes high wavenumber structures. Obvionsly this interpretation of critical wavenumbers is more general than that of the KS-theory. Especially the evolution of small scale patches to large scale patchiness is not excluded. The essential point is that we deal with a continous wavenumber spectrum. If we introduce box-like boundary conditions (e.g. $P(x,y) = 0$ for $x = 0;\ L_x$ and $y = 0;\ L_x$) we would find a discrete spectrum and then the interpretation of critical wavenumbers is the same as in the KS-model. But measurements of spatial phytoplankton spectra show clearly, that real spectra are continously (see e.g. DENMANN, PLATT 1975).

Dependent on the spatial structure of our advection field (3) the critical wavenumbers can vary rapidly with time Especially in the case of a deformation field ($\delta > 0$) k_1 tends to zero $k_1 \sim \sqrt{t}\ e^{-\sqrt{\delta} t}$, while in case of an eddy field ($\delta < 0$) k_1 and k_2 converges into constant values. In the former case the existence of stable patchiness seems to be rather improbable, while in the lather case a stable plankton patch can be expected.

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